Cost Minimization with Leontief technology

A firm has the production function $f(L, K) = min\{L, K\}$

1. Find the conditional demands.

2. Find the minimum cost function.

3. Using the minimum cost function from the previous item, find q^* that maximizes profits knowing that $p = w + r$.

Solution

1. The first thing we need to keep in mind is that the firm will demand $L = K$ since otherwise, it would be wasting money on some input because the *min* function takes the smaller of the two arguments. Knowing this, the function to minimize is: $wL + rK$ subject to a given production: $q = L = K$. Therefore we must have $L^c = K^c = q$.

2. To find the minimum cost, we replace the conditional demands in the costs:

$$wL^c + rK^c = wq + rq = q(r + w)$$

3. The profits are:
$$B = pq - q(r + w)$$

Knowing that $p = w + r$
$$q(w + r) - q(w + r) = 0$$

Therefore, maximum profits are achieved at any level of q.